

## AN ANALYSIS OF THE LIGHT CURVE OF PLUTO

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## ABSTRACT

The light curve of Pluto is analyzed in terms of a geometrical model consisting of bright and dark areas which are assumed to exhibit either a diffuse or a geometrical type of reflectivity. A Fourier analysis method is used to invert the observed light curve to obtain the longitudinal distribution of bright and dark areas for any combination of albedos selected for the two types of terrain. The analysis indicates that the light curve of Pluto can be readily understood in terms of a surface consisting of bright and dark areas. However, on the basis of the presently available photometric data, the existence or absence of limb-darkened material cannot be established.

## I. INTRODUCTION

The photometric observations of Kuiper (see Harris 1961), Walker and Hardie (1955), and Hardie (1965*a*) have established that the brightness of Pluto is variable with an amplitude of about 0.1 mag and a period of 6.39 days. Because of this definite periodicity, the light fluctuation has been attributed to rotation, and the asymmetry of the light curve has been associated with an uneven distribution of surface albedo.

The general problem of interpreting the light curve of a rotating body in terms of its shape and surface spottiness has been discussed in detail by Russell (1906). Although a unique solution for the spot distribution is not possible, useful information regarding the type of surface reflectivity and the general nature of the surface spottiness can be obtained. In the special case of a spherical body whose rotation axis is perpendicular to the line of sight, the problem may be treated analytically and the light curve inverted to obtain the longitudinal distribution of bright and dark surface regions. Moreover, in this special case, the presence or absence of odd and even Fourier terms in the observed light curve has implications concerning the type of surface reflectivity law. That is, for a geometrically reflecting surface, odd Fourier terms higher than  $n = 1$  do not appear in the observed light curve. Likewise, for a diffusely reflecting surface, even Fourier terms higher than  $n = 2$  are absent.

A body of Pluto's mass, radius, and rotational period cannot depart significantly from spherical form since the yielding strength of most plausible crust materials will be exceeded at depths greater than a few kilometers, and dynamical flattening must be negligible for Pluto. While there is, at present, no definite information regarding the orientation of the rotational axis, the relatively large amplitude of the light variation suggests that Pluto's axis is probably not far from perpendicular (Walker and Hardie 1955). Moderate departures from perpendicularity are not likely to have much effect on the light curve of a body which is spherical in shape. Tilting the rotational axis by a small angle has the effect of interchanging a small crescent-shaped area on the back hemisphere with one from the forward hemisphere. If the spots are fairly evenly distributed in latitude, then even for tilt angles as large as  $20^\circ$  the light contributed by the crescent will not be more than about 6 percent. For spots concentrated near the equator the effect would be even less. Accordingly, in the following analysis, we assume that the rotational axis is perpendicular to the line of sight.

## II. METHOD OF ANALYSIS

In order to allow for the presence of both odd and even Fourier terms in the light curve, it is assumed that any given surface element  $dS = R^2 \sin \theta d\theta d\phi$  can be described as consisting of a portion which reflects diffusely (reflected light  $\propto \cos^2 \gamma$ ) with normal albedo  $A$  and a portion which reflects geometrically ( $\propto \cos \gamma$ ) with normal albedo  $B$ , where  $\gamma$  is the angle between the outward normal of the surface element and the line of sight (see Lacis and Fix 1972).

The two reflectivity laws, while arbitrary, are fully capable of accounting for all Fourier terms in the light curve. Since Pluto is never observed far from zero phase angle, the geometrical reflectivity may refer to a Lommel-Seelinger type of surface or a surface with large-scale roughness where shadowing effects are important. The diffuse, or Lambert-law, reflectivity may be associated with a smooth diffusing surface or patches of snowlike material.

The fractional part of the surface element  $dS$  associated with diffusely reflecting material and having normal albedo  $A$  is designated by  $h$ . Taken over the entire surface,  $h(\phi, \theta)$ , having values between 0 and 1, gives the distribution of diffusely reflecting material on the planet. If this model is used, the theoretical light curve  $G(\phi_0)$ , which gives the brightness as a function of sub-Earth longitude  $\phi_0$ , is written

$$G(\phi_0) = B + \frac{1}{\pi} \int_{\phi_0 - \pi/2}^{\phi_0 + \pi/2} \int_0^\pi [A \sin^2 \theta \cos^2 (\phi - \phi_0) - B \sin \theta \cos (\phi - \phi_0)] h(\phi, \theta) \sin \theta d\theta d\phi. \quad (1a)$$

Since no information regarding the latitude distribution of surface materials appears in the light curve when the rotating body is viewed from its equatorial plane, we may integrate equation (1a) over the polar angle and express  $h(\phi)$  as a function of longitude only:

$$G(\phi_0) = B + \int_{\phi_0 - \pi/2}^{\phi_0 + \pi/2} \left[ \frac{4}{3\pi} A \cos^2 (\phi - \phi_0) - \frac{1}{2} B \cos (\phi - \phi_0) \right] h(\phi) d\phi. \quad (1b)$$

If we assume that the distribution of the diffusely reflecting material, expressed as a function of longitude, may be written as

$$h(\phi) = \sum_{n=0}^{\infty} [a_n \cos (n\phi) + b_n \sin (n\phi)], \quad (2)$$

then equation (1b) can be integrated to obtain the theoretical light-curve as a Fourier series in  $\phi_0$ .

For comparison, the photometric observations of Pluto have been fitted with the Fourier series

$$G'(\phi_0) = C_0 + \sum_{n=1}^{\infty} [C_n \cos (n\phi_0) + D_n \sin (n\phi_0)], \quad (3)$$

where  $C_0$  is the mean normal albedo of Pluto, a quantity which is not well known at present. Figure 1 shows the composite photometric data available for Pluto and includes the observations of Kuiper, Walker and Hardie, and Hardie. The composite data were taken from Hardie (1965b). The filled circles indicate observations made in the period 1953–1955, and the open circles indicate observations made in 1964. Figure 1 also shows the resulting Fourier series fit to the photometric data including terms up to  $n = 2$  (solid line) and  $n = 5$  (broken line). The Fourier coefficients  $C_n$  and  $D_n$  (normalized to  $C_0$ ), the combined Fourier coefficients  $(C_n^2 + D_n^2)^{1/2}/C_0$ , and the rms differences between the observed points and the  $n$ th Fourier representation are given in table 1.

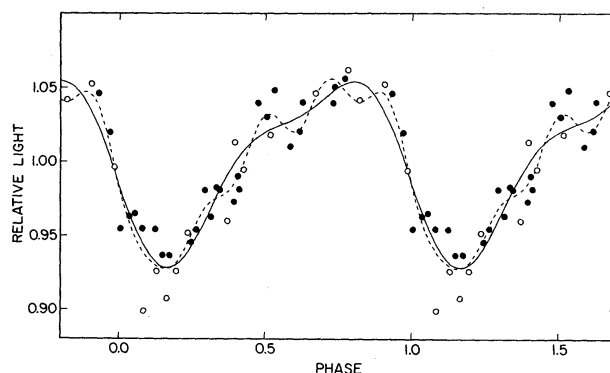
FIG. 1.—Fourier fits for  $n = 2$  and  $n = 5$  to the photometric observations of Pluto

TABLE 1  
THE COEFFICIENTS AND RESIDUALS OF A FOURIER  
FIT TO THE LIGHT-CURVE OF PLUTO

$n$	$C_n/C_0$	$D_n/C_0$	$(C_n^2 + D_n^2)^{1/2}/C_0$	rms/ $C_0$
0.....	1.00		1.00	$4.59 \times 10^{-2}$
1.....	$-1.70 \times 10^{-2}$	$-5.34 \times 10^{-2}$	$5.61 \times 10^{-2}$	$2.03 \times 10^{-2}$
2.....	$2.78 \times 10^{-3}$	$-1.83 \times 10^{-2}$	$1.85 \times 10^{-2}$	$1.60 \times 10^{-2}$
3.....	$1.39 \times 10^{-4}$	$-4.84 \times 10^{-3}$	$4.84 \times 10^{-3}$	$1.52 \times 10^{-2}$
4.....	$5.00 \times 10^{-5}$	$-4.69 \times 10^{-3}$	$6.86 \times 10^{-3}$	$1.49 \times 10^{-2}$
5.....	$-6.92 \times 10^{-5}$	$-1.57 \times 10^{-3}$	$7.10 \times 10^{-3}$	$1.37 \times 10^{-2}$

In comparing the Fourier coefficients of the observed light curve given in equation (3) with those obtained by integrating equation (1b), we find that for the  $\cos(n\phi_0)$  terms, the coefficients of  $h(\phi)$  are related to the observed light curve coefficients by

$$a_0 = \frac{C_0 - B}{\frac{2}{3}A - B}, \quad a_1 = \frac{C_1}{16A/9\pi - \frac{1}{4}\pi B}, \quad a_2 = \frac{3C_2}{A - B},$$

$$a_n = \frac{3\pi n(n^2 - 4)}{16A} (-1)^{(n+1)/2} C_n \quad (n = 3, 5, \dots),$$

$$a_n = \frac{n^2 - 1}{B} (-1)^{n/2} C_n \quad (n = 4, 6, \dots). \quad (4)$$

The same relationships also apply for the  $b_n$  and  $D_n$  coefficients of the  $\sin(n\phi_0)$  terms.

### III. DISCUSSION

Since the  $C_n$  and  $D_n$  coefficients of the observed light curve are known constants, the longitudinal distribution of the diffusely reflecting areas  $h(\phi)$  is determined for any specified combination of the albedos  $A$  and  $B$ . There are, however, large regions of the  $(A, B)$ -plane (see fig. 2) which must be excluded from consideration because they require physically impossible solutions where  $h(\phi)$  becomes greater than 1 or less than 0 over some range of longitudes. In general, the domain of permitted albedo combinations becomes smaller as more terms are included in the analysis. Also, there are two separate albedo regions which are consistent with the physical restrictions—one corresponding to bright

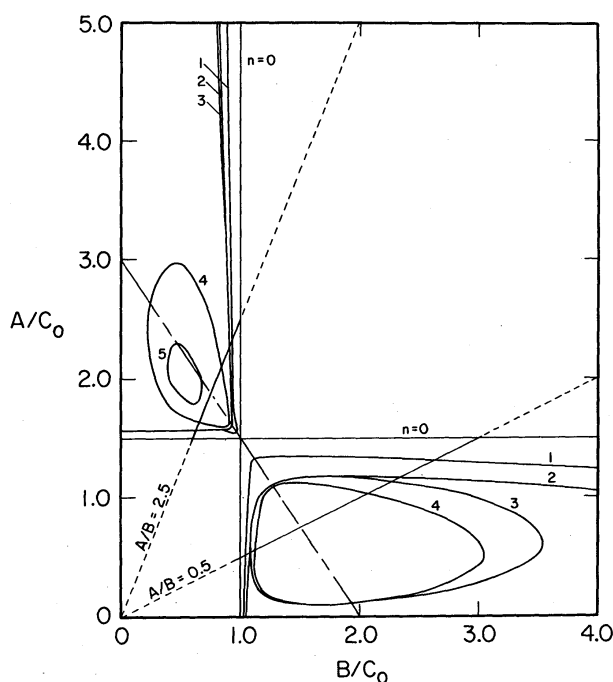


FIG. 2.—Domains of permitted albedo combinations corresponding to the specified number of Fourier terms used to approximate the light curve of Pluto. The normal albedos  $A$  and  $B$  designate diffusely and geometrically reflecting areas, respectively.

diffusely reflecting areas, the other to dark diffusely reflecting areas. It is impossible to differentiate between the two albedo regions on the basis of the observed light curve alone.

Because of observational scatter, the higher-order Fourier terms in the fit to Pluto's light curve become increasingly unreliable. Table 1 shows that we have little justification for including terms beyond  $n = 2$ . Including the  $n = 1$  and  $n = 2$  terms produces a marked improvement in the Fourier fit to the photometric data. This is indicated by the sizable reduction of the rms residuals. However, the addition of terms beyond  $n = 2$  does not appear to produce a significantly improved fit to the observed data.

Inspection of figure 2 shows that if only terms up to  $n = 2$  are retained, we cannot eliminate from consideration those models with  $A = 0$  or those with  $B = 0$ . That is, physically reasonable models of the surface of Pluto can be constructed which are entirely free of diffusely reflecting (limb-darkened) surface areas or which are entirely composed of such areas. Inspection of equations (4) shows that this result comes about because for terms up to  $n = 2$  the Fourier coefficients contain contributions from both geometrically and diffusely reflecting areas. It is only on the basis of odd terms  $n = 3$  and higher that we can verify the presence of diffusely reflecting areas, while the presence of geometrically reflecting areas can be ascertained only from even terms with  $n = 4$  or higher.

#### IV. CONCLUSION

On the basis of the presently available photometric observations of Pluto, surface models which are consistent with the observed light curve may be constructed entirely, or in part, from diffusely or geometrically reflecting material. Based on only the first two Fourier terms of the light curve, the limitations imposed on the allowed albedo combinations are not very strict. Although it is clear that a sizable difference in albedo

must exist between the brighter and darker areas, it cannot be ascertained whether the light variation is due to dark spots or to bright spots, or whether the spotted areas are large or relatively small.

It is possible that a more fruitful approach for distinguishing the principal type of surface material lies in searching for an opposition effect. It is known that many asteroids as well as the lunar surface (see, e.g., Gehrels, Coffeen, and Owings 1964) show a non-linear increase in brightness near opposition. Also, if the two types of surface areas have different polarization characteristics, then the change in polarization with phase, if it can be measured, could shed considerable light on the nature of the Pluto surface.

Otherwise, it is hoped that additional photometric observations of Pluto will improve the reliability of the higher-order Fourier terms to the point where it is possible to distinguish the definite presence or absence of the two types of reflecting areas considered. When this is the case, a more precise model of the surface of Pluto may be constructed.

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